A NUMERICAL SOLUTION FOR TIME DEPENDENT, MULTI-CHANNEL QUEUES AND AN APPLICATION TO THE ACUTE MINOR ILLNESS CLINIC, SILAS B. HAYS HOSPITAL FORT ORD, CALIFORNIA

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Monterey, California

THESIS

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TIME DEPENDENT, MULTI-CHANNEL
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CLINIC, SILAS B. HAYS HOSPITAL
FORT ORD, CALIFORNIA

by

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September 1974

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Approved for public release; distribution unlimited.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) A NUMERICAL SOLUTION FOR TIME DEPENDENT, MULTI-CHANNEL QUEUES AND AN APPLICATION TO THE ACUTE MINOR ILLNESS CLINIC, SILAS B. HAYS HOSPITAL FORT ORD, CALIFORNIA	5. TYPE OF REPORT & PERIOD COVERED Master's Thesis September 1974
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) David Lester VanAsdlen and Leonard Otto Wahlig	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	12. REPORT DATE September 1974
	13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Queue(s) Time Dependent Queue(s) Multi-Channel Queue(s)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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The study draws an analogy between the clinic and a multi-channel, time dependent queueing system. An analytic model is presented which uses time dependent arrivals and server schedules and determines expected queue size, expected delay and the probability of delays exceeding a given maximum length.

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1974

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ACKNOWLEDGEMENT

We would like to express our sincere appreciation to Professor Richard W. Butterworth. Without his analytic insight and computer expertise this study would not have been as significant or meaningful.

I. AMOSIST PROGRAM

A. HISTORY, PURPOSE, AND ORGANIZATION

In the late 1960's the United States Army became aware that with the termination of the draft a source of physicians for the Armed Services would be lost. This loss would certainly have profound effects upon the health services provided by the U.S. Army, especially in the areas of clinic care. In 1969 in order to help maintain adequate outpatient medical care in the face of manpower shortages, the Office of the Surgeon General formed the Automated Military Outpatient System (AMOS) Project.

From the work of Project AMOS has come the AMOSIST Program. This program has emerged due to the ability of Project AMOS to record the logic used by physicians in their diagnosis and treatment of minor illnesses so that a nonprofessional medical person (AMOSIST) with minimal training could safely treat certain types of medical problems. The AMOSIST would then become a provider of health care, extending the capabilities of the attending physicians. The AMOSIST Program is solely for ambulatory health care.

The AMOSIST Program is made up of two parts called the Triage and the AMIC. In the Triage part of the program an ambulatory patient is screened by an AMOSIST using a "Triage Note" (diagnostic decision tree questionnaire) in order to determine to which hospital clinic the patient should be sent. The Acute Minor Illness Clinic (AMIC) is staffed by AMOSISTs and physicians (AMOSIST MDs) who treat people screened for the AMIC by the Triage. The "Triage Note" will specify whether the

patient is to see an AMOSIST or an AMOSIST MD, but a patient may request to see only an AMOSIST MD if he desires.

AMOSISTs are always under the supervision of a physician, who sees patients sent directly from the Triage Section and patients referred from the various AMOSISTs. An AMOSIST may refer to a physician if he does not feel sure of his diagnosis or is faced with an unfamiliar complaint. The physician may merely confer with the AMOSIST verbally or he may go to the AMOSIST's room to see the patient himself.

In order to make the program successful, the criteria for AMOSISTs are very stringent. Upon selection, these personnel are given two weeks didactic training at Fort Sam Houston, Texas, followed by about ten weeks on the job training as stated in Reference 1. Physicians attend the two weeks of didactic training and may then return to their facilities to help ready the clinic.

At present the AMOSIST Program has been adopted in over 20 Army Medical Facilities. Adoption of this program at a particular facility is decided after studies have been done to ensure the program's feasibility.

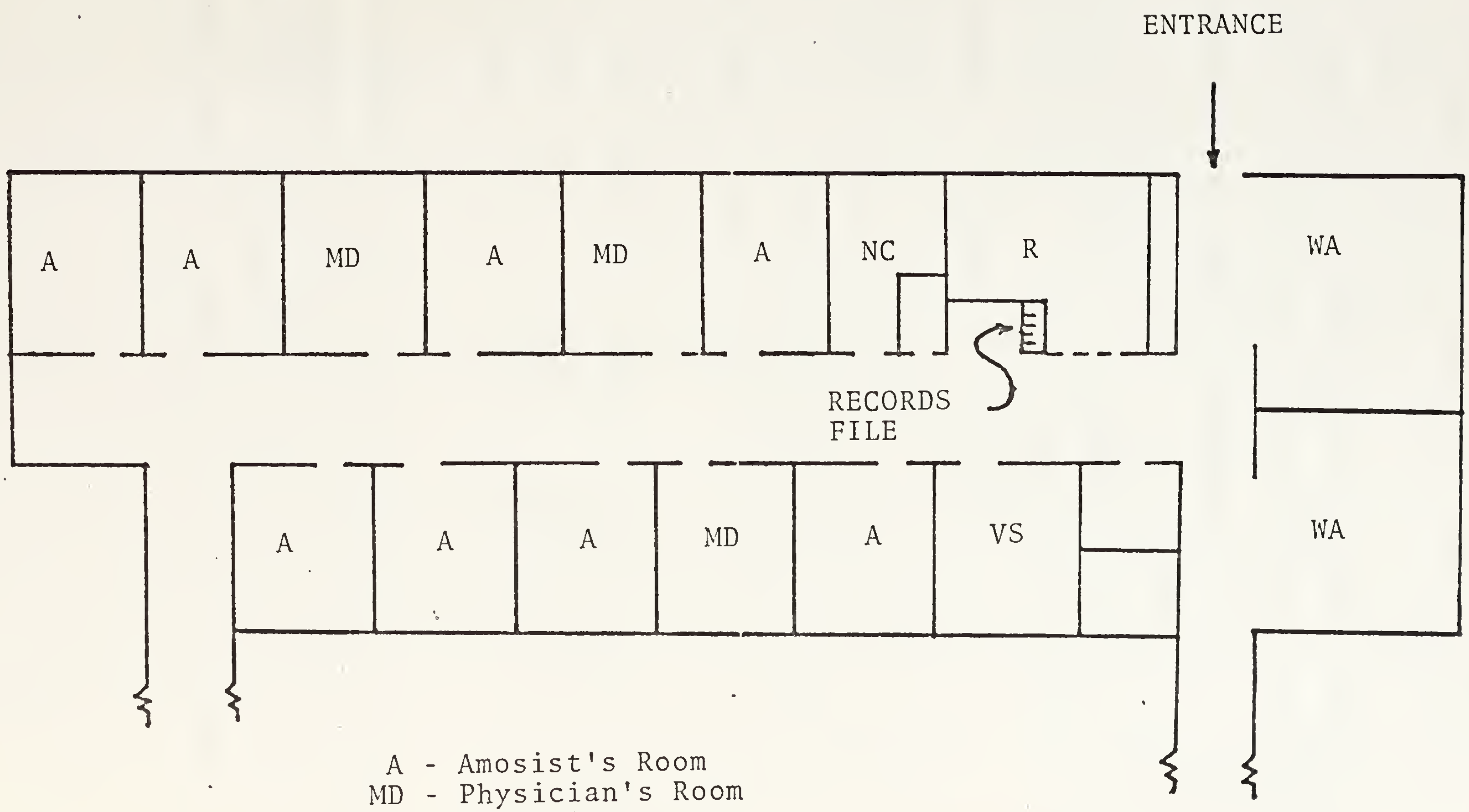
B. IMPLEMENTATION AT SILAS B. HAYS HOSPITAL

In December, 1973, the AMOSIST Program was instituted at Silas B. Hays Hospital, Fort Ord, California. Figure 1 shows the physical layout of the AMIC.

The Triage is physically separate from the AMIC and is located next to the patient records room. Patients who are directed (Triaged) to the AMIC are logged in at the AMIC reception desk and have necessary vital signs taken by a Nursing Assistant. The patient's folder is then placed

FIGURE 1

AMIC FLOORPLAN



A - Amosist's Room
MD - Physician's Room
R - Reception
WA - Waiting Area
VS - Vital Signs

in either a physician or AMOSIST rack depending on the "Triage Note" attached. The patient is seen when a server becomes available. Order of service is first in, first out.

During a treatment, a chaperone may be necessary. Nursing Assistants or even AMOSISTs are used for this. If referral to a physician is necessary, the AMOSIST will look for an available physician. He may have to wait if all physicians are busy. When a physician is located, either a verbal referral will occur or the physician will go with the AMOSIST to examine the patient. In either case the effect is the same, namely that two persons are occupied with one patient.

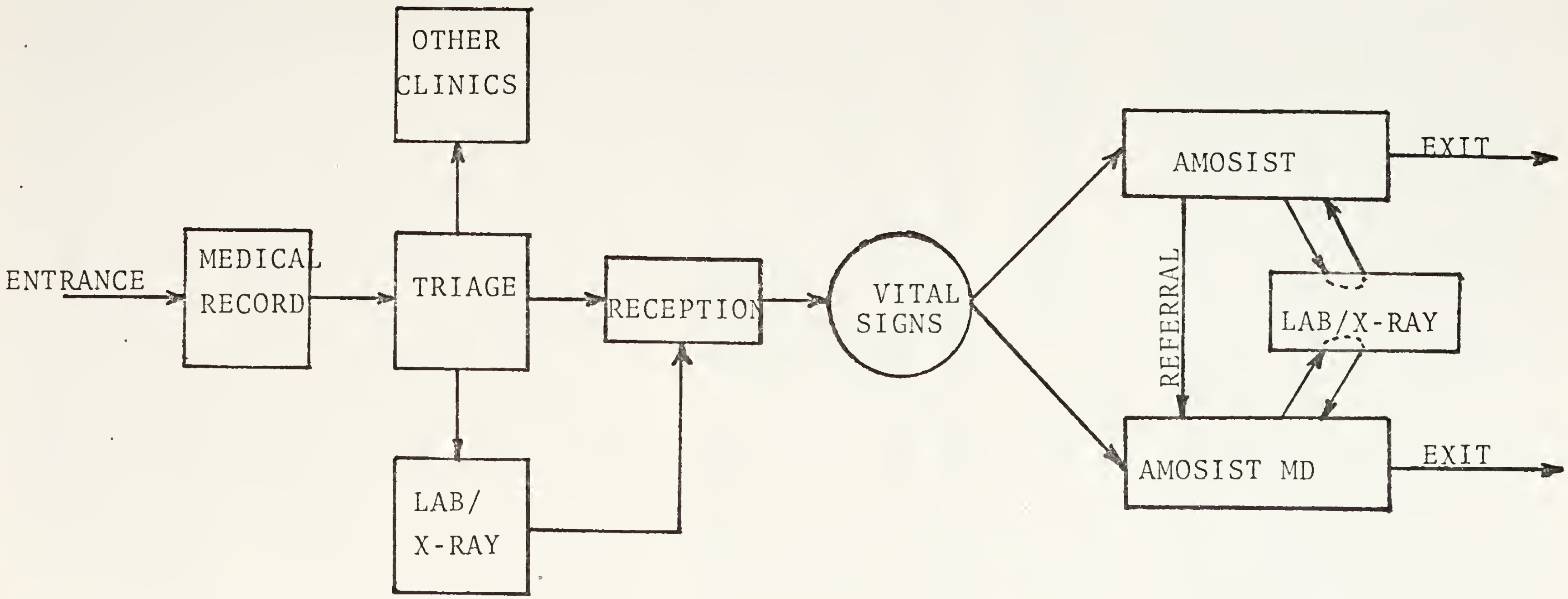
A small percentage of patients come to the AMIC just for prescription refills. Their records are placed in a separate rack. Refill patients are given a priority by the physicians and wait in a separate area from the other patients. Their service times are normally very much shorter than those for other patients.

Another small percentage of patients are sent from the AMIC to the X-RAY Department or to the Laboratory. Upon their return to the AMIC, they are given priority by the AMOSIST or physician who originally saw them.

Figure 2 presents a block diagram of the system just described. It can be determined at this point that, with the exception of referrals from the AMOSISTs to physicians, the system can be viewed as two separate multi-channel queues. The analytical model will be discussed in another Section.

FIGURE 2

BLOCK DIAGRAM OF AMOSIST PROGRAM



II. AMIC DATA

In order to develop an analytical model capable of analyzing and predicting the system behavior of the AMIC at Silas B. Hays Hospital it was necessary to establish a data base. To keep the modeling of the AMIC in a workable form, all statistics related to prescription refills and patients sent from the AMIC to X-RAY or LAB were disregarded. Also, complete statistics were confined to weekdays when patient load was the heaviest.

A. PERSONNEL STAFFING AND SCHEDULING

During this study, there were four physicians available, one being responsible for the clinic's medical administration as well as seeing patients. There were 11 AMOSISTs available who either saw patients or assisted in the Triage. An additional non-medical enlisted person was the NCOIC. The support staff was made up of four Nursing Assistants and three receptionists.

The AMIC was open to receive patients from 0745 to 2330 hours, with active duty personnel having priority prior to 1000 hours. The day shift of personnel work until 1630 while the night shift worked from 1530 to 2330 hours. During weekdays, three AMOSIST MDs worked on the day shift and one at night. Since the AMIC was tasked with sending a physician to the stockade each morning, this meant that at most two physicians would be available during the mornings.

On Monday, the heaviest day, seven to eight AMOSISTs would be scheduled to work the day shift and on the other weekdays, five to six. At night, two AMOSISTs were scheduled on all weekdays.

B. PATIENT ARRIVAL PATTERN

In order to get an adequate estimate of the patient arrival pattern in the AMIC, log-in-sheets for February to May 1974 were reviewed. Each month's arrivals were recorded for each hour of the day and for each day of the week. These arrival figures were averaged together to give a standard week. Holidays and days affected by holidays were left out of the calculations since they were unusual in value and not representative of an average day. For periods that did not include a full hour period, the figures were adjusted to an hourly figure in order to make them comparative for statistical purposes. Table I summarizes the arrival figures for an average week. Each day's average was based on 17 weeks of data.

Each day of the week exhibits approximately the same shape curve for arrival patterns over the length of the day and differs only in the average daily total number of arrivals. This allowed arrival figures to be pooled together giving one average weekday arrival pattern as depicted in Figure 3. Different patient volumes could then be achieved by merely moving the entire curve up or down to produce any desired patient volume.

C. AMOSIST MD SERVICE TIMES

There was no procedure within the AMIC to recover the length of time that an AMOSIST MD spends with a patient or with an AMOSIST on a referral. To estimate these times, approximately 75 actual observations of patient encounters with AMOSIST MDs were made during randomly selected days over a three week period. The results of these observations showed that an AMOSIST MD spends approximately 13.55 minutes with his own patients and about 4.34 minutes of his time with an AMOSIST on a referral.

TABLE I

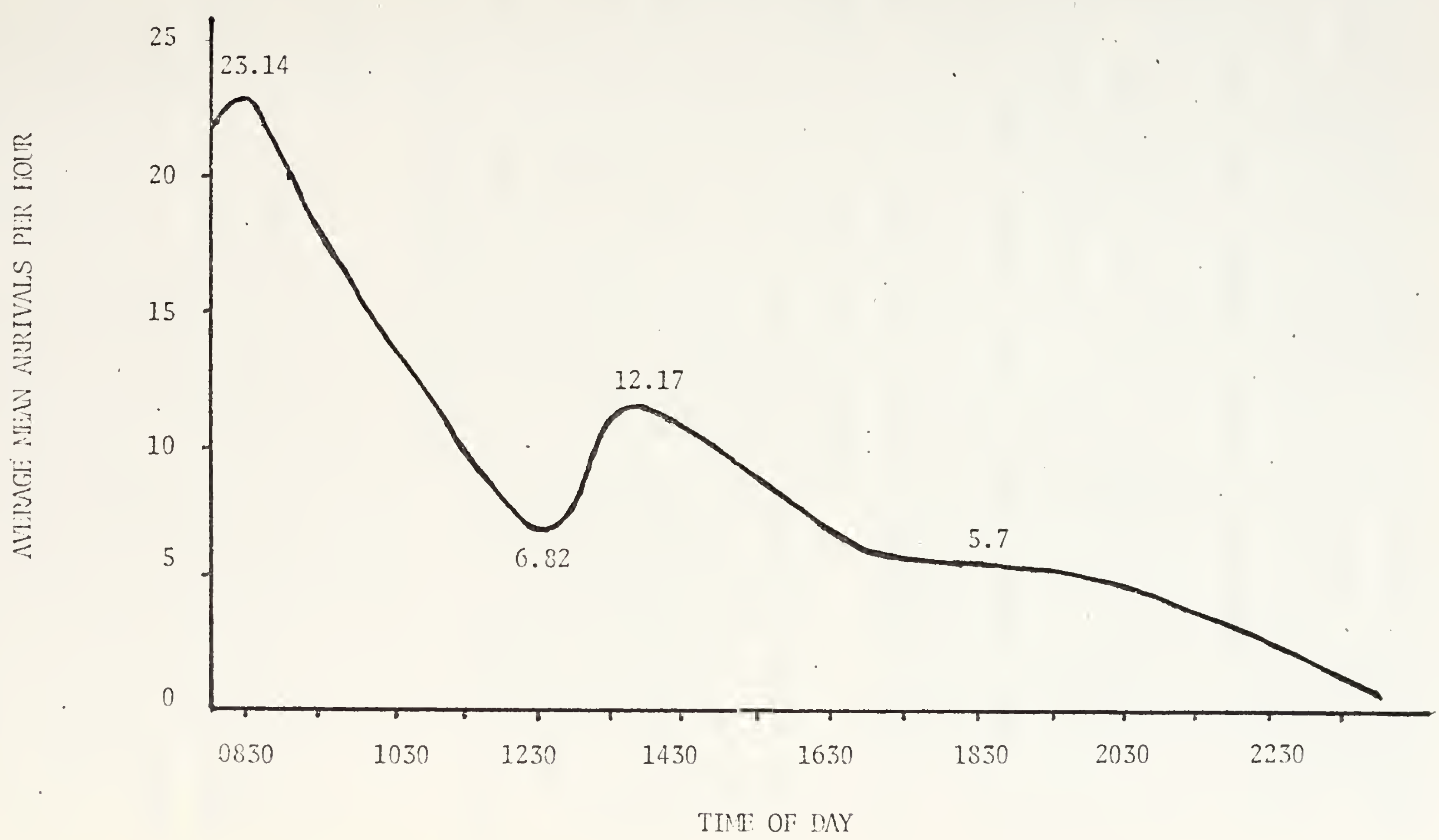
ARRIVALS PER HOUR FOR AVERAGE WEEK

	MON.	TUES.	WED.	THUR.	FRI.
0745 - 0800*	18.48	15.24	10.36	9.88	12.88
0800 - 0900	24.69	23.50	25.12	22.00	20.67
0900 - 1000	21.75	20.81	17.94	16.65	14.94
1000 - 1100	17.50	13.56	16.47	13.76	12.56
1100 - 1200	10.25	9.00	8.41	8.71	9.67
1200 - 1300	7.88	7.56	6.12	5.94	6.72
1300 - 1400	11.31	13.50	12.47	12.65	11.06
1400 - 1500	10.19	11.13	9.71	11.82	9.44
1500 - 1600	9.06	8.31	7.65	7.82	8.50
1600 - 1700	8.06	5.69	6.24	6.82	6.11
1700 - 1800	7.31	6.56	5.12	4.76	4.33
1800 - 1900	5.44	5.94	6.71	6.47	4.39
1900 - 2000	6.25	6.69	5.71	5.65	4.67
2000 - 2100	5.13	5.38	4.47	4.76	4.33
2100 - 2200	3.56	3.88	3.06	3.24	2.83
2200 - 2300	2.00	2.50	2.55	3.11	3.22
2300 - 2330*	.50	.76	.88	.88	1.10

* ADJUSTED TO HOURLY AVERAGE

FIGURE 3

TYPICAL WEEKDAY ARRIVAL PATTERN



D. AMOSIST SERVICE TIMES

The service times for each AMOSIST were broken down into referral and non-referral categories just as for the physicians. The actual times spent with patients for each individual AMOSIST were taken from the Data Collection Sheets (computer generated questionnaire used by AMOSIST to diagnose a patient's complaint) which are kept on file for each AMOSIST. These sheets also show by a physician's signature or an appropriate box being checked when a referral was made. After analysis of these actual patient encounter times, an average service time (time from start of one patient to start of next patient) was derived for each AMOSIST. In all but one case, at least 40 encounters were used to compute average "service times" for an AMOSIST.

The results of this data indicated that the average service time for a non-referred patient was 17.96 minutes while the referred patient's service time was 23.85 minutes. The actual patient encounter times for the AMOSISTs compared quite favorably with results published in a recent AMOS Project Progress Report (Reference 2). In addition, an average referral rate for an AMOSIST was found to be 30.93%. The above estimates were based on about 400 AMOSIST's encounters.

E. OTHER STATISTICS

Since the AMIC is a system that generates patients to two different types of servers, it was necessary to find the percentage of people who were either triaged to see an AMOSIST MD or requested to see an AMOSIST MD. This type of data was not readily available from the records of the AMIC. For selected weeks the Data Collection Sheets for all AMOSISTs were totaled for certain days. This figure showed the total patients

triaged to an AMOSIST who actually saw an AMOSIST on a given day. This was subtracted from the total patients entering the AMIC, as shown on the log-in-sheets, to give the number of patients seeing an AMOSIST MD directly. This figure showed that approximately 55% of the patients were sent directly to the AMOSIST MD versus 45% for the AMOSIST. This estimate originated from approximately 1000 cases.

Since it was decided to start the analytical model at 0800 in the morning, an estimate of the average number of people in the waiting area prior to the first patient being seen was required. The arrival information previously gathered for February to May, 1974, was used to find an estimate of 11 people waiting to see a server prior to the first service time.

F. INTERPRETATION OF DATA

It would be inaccurate to say at this point that the AMIC at Silas B. Hays Hospital can be fully represented with the aforementioned set of numbers. Any system that is composed of people and is dependent upon their interactions can probably never be completely and accurately modeled.

The data gathered on the system were required by the analytical approach to the AMIC model. Also, the analytical model is a long run representation and average figures should not be expected to predict exactly what the situation will be like tomorrow. Some variation will almost always occur.

Each system has its own inherent characteristics that may not be readily discernible at first glance. The AMIC at Silas B. Hays is no exception. The stockade commitment in the mornings requiring a physician

from the AMIC, the priority until 1000 for active duty personnel giving an uneven arrival pattern, and the makeup of the population that the AMIC serves, are things that may not be common to any other AMIC. Each system must be judged on its own performance in the face of its individual problems and not be compared using "raw data" to other systems which outwardly appear the same.

The analytic model output to be presented pertain to Silas B. Hays Hospital as it was operating during the time of this study and uses data taken at a particular period of time. No doubt as the system changes in time certain statistics may need revision. The greatest use of this study will be the chance to see the results of changes, either proposed or actual, and predict their effect upon the entire system.

Table II summarizes the data that has been presented in this Section and which will be used as the data base for the analytic model.

TABLE II

DATA SUMMARY

AMOSIST MD:

MEAN SERVICE (DIRECT)	13.55 min.
MEAN SERVICE (REFERRAL)	4.34 min.

AMOSIST:

MEAN SERVICE (NON-REFERRAL)	17.96 min.
MEAN SERVICE (REFERRAL)	23.83 min.
REFERRAL RATE	30.93%

PROPORTION OF DIRECT MD PATIENTS	55.00%
MEAN ARRIVALS PRIOR TO FIRST SERVICE	11

AVERAGE ARRIVALS PER WEEKDAY:

MON.	TUES.	WED.	THURS.	FRI.
156	149	145	140	127

STAFFING:

AMOSIST MD	4
AMOSIST	11
NURSING ASST.	4
NCOIC	1
RECEPTIONIST	3

III. THE AMIC AS A QUEUEING SYSTEM

During the course of data gathering and research, the similarities of the Acute Minor Illness Clinic to a multi-channel queueing system became apparent. In the actual system the arrivals follow an inhomogeneous pattern throughout the course of any day. Once in the system a patient is directed to an AMOSIST or an AMOSIST MD and then waits for the first available server. When a server is free, the earliest arrival waiting is treated and released. The major complication in modeling the actual system is the interference in the AMOSIST MD queue caused by referrals from AMOSISTs. However, if the two queues (AMOSIST and AMOSIST MD) can be isolated, two independent queueing problems can be formulated.

The AMOSIST referrals cause an interference in the AMOSIST MD queue because the referred patient occupies an AMOSIST MD with an additional service. This however, is the only effect because an AMOSIST remains occupied while the referral takes place. Thus the problem can be simplified by determining an appropriate augmentation to the AMOSIST MD arrival stream to handle those patients referred by AMOSISTs. These additional arrivals can be considered as "virtual" arrivals - i.e. persons who will eventually occupy the AMOSIST MD's time but who initially arrive to see the AMOSIST.

The referral rate from AMOSIST to AMOSIST MD is taken as a percentage of the AMOSIST arrival stream. The number of arrivals thus computed was then added to the AMOSIST MD arrival stream. The original clinic system can then be viewed as two distinct multi-channel queueing systems.

In order to use the data presented in Section II as an input to the analytic model, it was necessary to calculate certain new statistics. The hourly arrivals as well as a single average service time for an AMOSIST and an AMOSIST MD were computed.

For a given hourly arrival figure, the product of this number and the average percentage seeing an AMOSIST gives the hourly arrivals for the AMOSIST. The hourly arrivals for an AMOSIST MD is the product of total hourly arrivals and the percentage seeing the physician directly added to the product of the referral rate and the hourly arrivals for an AMOSIST.

The analytic model recognizes only one average service time for an AMOSIST or an AMOSIST MD. For both servers, Section II gives two different types of service times. For the AMOSIST, it was referral and non-referral service times. The AMOSIST MD had either direct patient or referral service times. The percentage of each type of service given by an AMOSIST or AMOSIST MD was computed. The product of this percentage and the appropriate average service time added over types of service gives an effective average service time for the AMOSIST and AMOSIST MD.

IV. THE MODEL

A. GENERAL DESCRIPTION

In order to analyze the two queueing systems, a mathematical model was developed. Using the inhomogeneous arrival stream and server schedules as input, the model computes the probability distribution of system size throughout the day. The model then computes various measures of system performance at specified times of the day. The model treats the two queues as separate systems and during each day solves first the AMOSIST system and then the AMOSIST MD system.

The significant problem in the formulation of the model was that all rates and schedules of servers were functions of time. There is no known analytical method of solution for the time dependent multi-server queueing system. Katadare and Kaufman (Reference 3) have advanced a Method of Successive Approximate Transient Solutions (MOSATS) for numerical analysis of a single channel time dependent system and much of their method has logical extensions to the multi-channel system. The approach calls for discretizing the time axis into segments over which the parameters remain constant. Then a transient solution is computed for each segment successively. The method was expanded to multiple channel queues and made efficient for real time computations for its application to this problem. The details are explained more fully below.

B. MODEL ASSUMPTIONS

The model makes several key simplifying assumptions. These will be listed and explained briefly here and referred to in subsequent sections.

(A1) Arrivals form an inhomogeneous Poisson Process.

(A2) Services are exponentially distributed and independent of the arrival stream.

(A3) At most three arrivals and three departures can occur in any micro time step. A micro time step is a sub-interval of the periods in which parameters remain constant; it is used in the computation of state probabilities.

(A4) No patient can both arrive and depart within the same micro time step. Patients leaving during a micro time step are from those present at the beginning of the step. This assumption has the effect of simplifying the calculations somewhat.

C. MODEL OPERATION

In order to adequately describe the operation of the model, a general description of the computations for solving a day's clinic operation is presented here. Mathematical amplifications are presented with the appropriate steps where necessary. Output and interpretation are discussed later.

1. Initial State Probability Vector

The probability distribution of system size at the day's beginning is Poisson with a mean given by the user. Specifically,

$$P_n(0) = \frac{p^n e^{-p}}{n!}, \quad n = 0,1,2,\ldots,N$$

where $P_n(t)$ is defined to be the probability that system size is n at time t and p is the average number of early arrivals for the given day. To ensure that no probability mass was lost by limiting system size to N, the model places all remaining mass in the probability of largest system size. That is,

$$P_N(0) = 1.0 - \sum_{n=0}^{N-1} P_n(0).$$

In all applications this value was zero. System size (N) used in the model was 15 for computational efficiency. This restriction is not severe because in light of the two queue concept, it represents an overall system size of 30.

2. Time Step Computations

The model next determines the first period of time during which all parameters (arrival rate and number of servers) are constant. It should be noted that both arrival rates and server schedules are specified as step functions. Within this interval, subsequently referred to as the macro interval, time dependence is removed. Using (A3) and (A4) this macro interval is subdivided into micro intervals such that the assumptions are valid. Specifically,

$$\delta = \mathrm{Min}\left(\frac{1.25}{\lambda}, \frac{1.25}{c\mu}\right)$$

where δ is the length of the micro interval, λ is the appropriate arrival rate, c is the number of servers and μ is the service rate per server. The length of the macro interval is then divided by δ to determine the number of "steps" which will be necessary to compute the state probability vector at the end of the period. The constant (1.25) used in the δ computation was experimentally determined as that figure which gave the best approximation to a correct steady state solution for a time independent system.

3. Transition Probability Matrix

In order to determine the probability of entering a given state from an initial state in a δ step, a matrix of transition probabilities [Q] is next determined. The entries in this matrix are defined as

$$Q_{k,n}(t) = \Pr\ (\text{system size at } t \text{ is } n \text{ given system size at } t-\delta \text{ was } k)$$

Further define

$$Q_{k,i,j}(t) = \Pr \text{ (i arrivals and j departures in } \delta \text{ given system size at } t-\delta \text{ was k).}$$

Then we have approximately that

$$Q_{k,n}(t) \doteq \sum_{\substack{i \leq 3 \\ j \leq 3 \\ k+i-j=n}} Q_{k,i,j}(t)$$

To determine the $Q_{k,n}$ two more definitions will be necessary:

$$X(i) = \Pr \text{ (i arrivals in a } \delta \text{ step) } i = 0,1,2,3$$

$$Y(j,k) = \Pr \text{ (j departures in a } \delta \text{ step given k present at the beginning of the } \delta \text{ step) } j = 0,1,2,3.$$

It is clear that the probability of an arrival is independent of system state and/or number of servers. In fact, by (A1), the probability of i arrivals in δ is

$$X(i) = (\lambda\delta)^i e^{-\lambda\delta}/i! \quad , \; i = 0,1,2,3$$

By (A3) the number of departures in δ is limited to three. This restriction defines three distinct cases for the probability of a departure based on system size and number of servers. These cases are first discussed and then the combination of arrival probabilities and departure probabilities to determine the $Q_{k,n}(t)$ entries is discussed.

a. System Size Less Than c

An entering system size less than the number of servers implies that not all servers are occupied and a binomial probability of departure exists. Specifically,

$$Y(j,k) = \binom{k}{j} (1.0-e^{-\mu\delta})^j (e^{-\mu\delta})^{k-j} \quad , \; j = 0,1,2,3 \text{ and } k \geq j.$$

b. System Size Greater Than c+2

When the beginning system size exceeds the number of servers by two or more, the system is saturated and no server becomes idle during the period for cases of at most three departures. The probability of a departure is therefore k-independent. Specifically,

$$Y(j,k) = (c\mu\delta)^j e^{-c\mu\delta}/j! \quad , j = 0,1,2,3 \text{ and } k > c+2.$$

c. System Size Between c and c+2

When the beginning system size is such that all servers are occupied but also might become idle at some point during the time interval, the exponential assumption (A2) must be interpreted properly to account for the possibility of idle servers. If, in fact, the servers become idle during δ , the distribution of inter-departure time is 2-Erlang followed by 3-Erlang, etc. depending on the specific number of servers becoming idle. Looking more closely at the specific cases involved, it can be shown that:

$$\text{for } k = c+1, \; Y(0) = e^{-c\mu\delta}$$

$$Y(1) = (c\mu\delta)\, e^{-c\mu\delta}$$

$$Y(2) = c^2(e^{-(c-1)\mu\delta})(1-e^{-\mu\delta} - \mu\delta\, e^{-\mu\delta})$$

$$Y(3) = 1.0 - \sum_{j=0}^{2} Y(j)$$

and for k = c+2,

$$Y(0) = e^{-c\mu\delta}$$

$$Y(1) = (c\mu\delta)\, e^{-c\mu\delta}$$

$$Y(2) = (c\mu\delta)^2 e^{-c\mu\delta}/2!$$

$$Y(3) = 1.0 - \sum_{j=0}^{2} Y(j)$$

Finally, the entries in the transition probability matrix may be determined. By (A4) arrivals and departures are independent events and thus $Q_{k,i,j} = X(i)\ Y(j,k)$. It should be emphasized that the effects of k (entering state) are in the Y(j) term.

A necessary property of transition probability matrices in general is that the row elements sum to 1.0. That is, given any beginning state, the system must be in some state at the end of the interval. By (A3) the model discards any tail probability beyond three arrivals and/or three departures. To correct for this assumption and to ensure that correct row sums are obtained, the convention of placing all remaining probability mass in the X(3) and Y(3,k) terms was adopted, except in those cases where the entering state was less than three. In those cases the remaining probability was placed in the highest possible number of departures consistent with entering system size. (Recall (A4)).

4. <u>The Interval Computation</u>

Having determined the δ step transition probability matrix, it is necessary to show the computation used to determine follow-on state probabilities. It can be readily seen that:

for $P(t) = (P_0(t), P_1(t), \ldots, P_N(t))$,

$$P(t+\delta) = P(t)[Q]$$

$$P(t+2\delta) = P(t+\delta)[Q] = P(t)[Q][Q] = P(t)[Q]^2$$

$$\vdots$$

$$P(t+i\delta) = P(t)[Q]^i \quad ,$$

where i is the previously computed number of δ steps contained in the macro interval under consideration. (See IV.C.2 above). Within the model, i is taken to be a power of two for computational efficiency.

5. Iterations

The results of the preceding computations are state probability vectors for the end of each macro interval. This state probability vector replaces the initial probability vector (1) and the computations (2 through 4) are repeated iteratively until a predetermined end of day is reached. The model then returns to the second arrival rate and server schedule (AMOSIST MD) and repeats the entire procedure until the predetermined end of day is again reached. This then represents the end of one day's clinic operations.

D. STATISTICS

1. Identified

It was initially recognized that probabilities of system size and delay would be necessary if the model were to be of any practical value to the user. After considering the myriad of available interpretive tools, the list was ultimately narrowed to eight distinct measures of the system which are determined at each time the user desires a report. These statistics are defined and discussed separately in the following sections.

a. Expected System Size is computed as

$$\sum_{n=0}^{N} nP_n(t).$$

b. Standard Deviation of System Size is computed as

$$\sqrt{\sum_{n=0}^{N} n^2P_n(t) - \left(\sum_{n=0}^{N} nP_n(t)\right)^2} \,.$$

c. The probability a server is busy at t is

$$\rho = \left(\sum_{n=0}^{c-1} nP_n(t)+c\sum_{n=c}^{N} P_n(t)\right)/c.$$

It should be noted that this probability is analagous to the standard utilization determination of queueing theory and is hereafter referred to as utilization (or ρ).

d. The expected size of the queue at t is

$$\sum_{n=0}^{N} nP_n(t) - \sum_{n=0}^{c-1} nP_n(t) - c\sum_{n=c}^{N} P_n(t) .$$

e. The standard deviation of queue size at t is

$$\sqrt{\sum_{n=c}^{N} (n-c)^2 P_n(t)}.$$

f. The probability an arrival at t encounters no delay is

$$\sum_{n=0}^{c-1} P_n(t) .$$

g. The probability an arrival at t encounters delay in excess of d minutes is

$$\sum_{n=c}^{N} \left\{ \sum_{k=0}^{N-c} \frac{(e^{-c\mu d})(c\mu d)^k}{k!} \right\} P_n(t).$$

h. The expected delay for an arrival at t is

$$\frac{\sum_{n=0}^{N} nP_n(t) - \sum_{n=0}^{c-1} nP_n(t) - c\sum_{n=c}^{N} P_n(t) + \sum_{n=c}^{N} P_n(t)}{c\mu} .$$

2. <u>Interpretation</u>

It is worthwhile to note that because of the augmentation of the AMOSIST MD arrival rate it is necessary to adjust certain of the above statistics to remove the effect of the "virtual" arrivals. Statistics a,b,d and e are therefore interpreted as follows: a and d: the expectation of system and queue size are adjusted by a factor of p where p represents the percentage of the arrival stream which are actual AMOSIST MD arrivals; b and e: the standard deviations are computed as

$$\sqrt{pqE(X) + p^2V(X)}$$

where p is as above, q is (1-p) and X represents the computed and unadjusted figures for b and c.

E. IBM 360/67 APPLICATION

1. General

The above model has been programmed in G-Level Standard Fortran for implementation on the IBM 360/67 under OS and for the CP/CMS time sharing system at Naval Postgraduate School. The general flow chart of the model is included as Figure 4. The model successfully reflects the actual system operation of the AMIC at Silas B. Hays Hospital. It has been incorporated in an interactive time sharing mode and through the use of portable terminal equipment has been demonstrated to hospital and clinic administrators.

2. Input

The model is sufficiently general to allow user control of the following input parameters:

a. Volume.

The user may specify any number of arrivals desired in the clinic for a given day. The model interprets this volume as a scale factor for the standard arrival rate step function.

b. Service Time

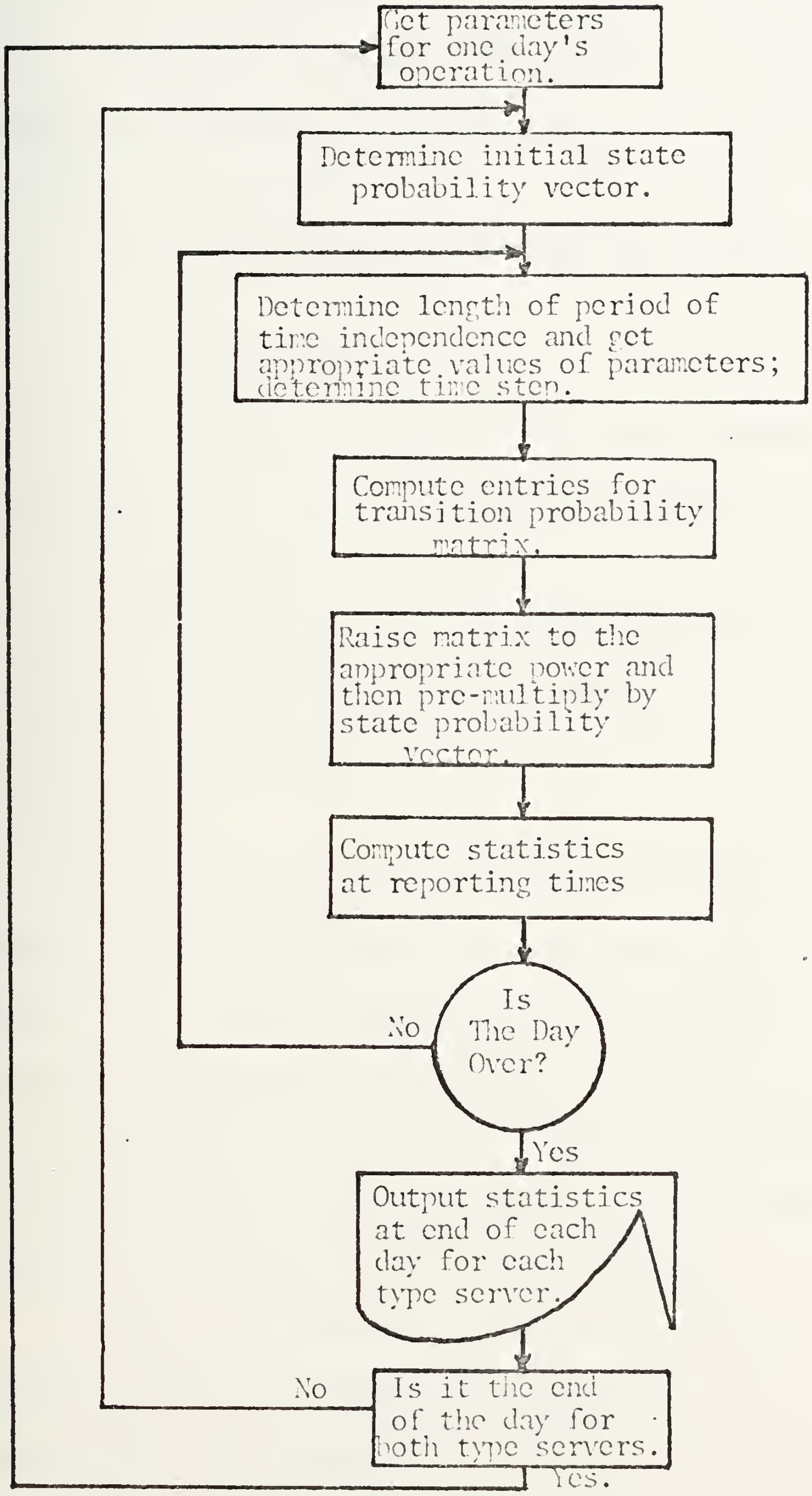
The user may vary the mean service time for either type of server and also for referral and non-referral service.

c. Schedules

The user may specify schedules for each type of server and may have the number of each type server on duty change as many as 16 times throughout the day.

FIGURE 4

FLOWCHART



d. Percent of Arrival Stream

The user may specify that percentage of the arrival stream which sees the doctor directly.

e. Referral Rate

The user may specify that percentage of patients which will ultimately see a doctor through referral from an AMOSIST.

f. Early Arrivals

The user may specify the number of arrivals before the system opens.

g. Reports Desired

The user may specify up to 16 times throughout the day at which he would like statistics computed and displayed.

h. Delay Cutoff

The user may specify the number of minutes for which he would like the probability of delay in excess of that figure computed. (the "d" in IV.D.1.g above).

3. Output

Although eight statistics are generated within the model, it is apparent that not all of these have meaning for the non statistically oriented user. For this reason only five of the eight are normally displayed in interactive user sessions. These five are time of the report, utilization, expected size of the queue, expected delay encountered and the probability of delay in excess of the user supplied delay cutoff. The entire list of statistics is printed off-line.

Tables 3 and 4 show the statistics generated for a typical weekday for AMOSIST and AMOSIST MD respectively. The server schedules are also shown. The normal method of gathering statistics is to compute

at one minute past each hour to capture conditions which will be in force for the coming period. That is, because most changes in schedule and arrival rate step functions occur on the hour, the statistics are taken during the following period so that the conditions which will attempt to clear the system will be in force.

TABLE III

AMOSIST QUEUE STATISTICS FOR A TYPICAL WEEKDAY

L	σ_L	ρ	L_q	σ_{L_q}	$P(D=0)$	$P(D>15)$	$E(D)$	Time
6.0333	3.6306	0.8634	2.5797	3.0681	0.2940	0.4577	15.9726	0901
4.4304	3.2240	0.7483	1.4371	2.4513	0.4902	0.2861	9.4642	1001
2.9957	2.4316	0.7048	0.8812	1.8273	0.5154	0.2534	8.8527	1101
1.9830	1.8713	0.5386	0.3672	1.1466	0.7183	0.1300	4.2059	1201
1.3474	1.3310	0.3236	0.0529	0.4109	0.9404	0.0184	0.5469	1301
1.9065	1.4686	0.4540	0.0904	0.4562	0.8723	0.0365	1.0604	1401
1.6577	1.3535	0.4007	0.0550	0.3468	0.9103	0.0240	0.7032	1501
1.2925	1.1456	0.2577	0.0038	0.0821	0.9888	0.0015	0.0585	1601
0.9823	1.2018	0.4022	0.1779	0.5904	0.7433	0.1194	4.2250	1701
0.8537	1.1556	0.3532	0.1473	0.5690	0.7882	0.0976	3.4909	1801
0.8383	1.1393	0.3491	0.1401	0.5552	0.7928	0.0948	3.3769	1901
0.8286	1.1318	0.3458	0.1370	0.5481	0.7955	0.0934	3.3206	2001
0.6645	0.9965	0.2880	0.0885	0.4337	0.8456	0.0681	2.3623	2101
0.4159	0.7467	0.1923	0.0313	0.2444	0.9197	0.0329	1.0842	2201
0.3041	0.6100	0.1454	0.0132	0.1462	0.9502	0.0193	0.6129	2301
0.0918	0.3142	0.0456	0.0007	0.0323	0.9937	0.0022	0.0679	2400

AMOSIST Schedule

4 0800-1100
3 1100-1300
4 1300-1530
5 1530-1630
2 1630-2330

LEGEND

L Expected System Size
σ_L Standard Deviation of System Size
ρ Probability a Server is Busy
L_q Expected Queue Size
σ_{L_q} Standard Deviation of Queue Size
$P(D=0)$ Probability No Delay
$P(D>15)$ Probability Delay>15 min.
$E(D)$ Expected Delay

TABLE IV

AMOSIST MD QUEUE STATISTICS FOR A TYPICAL WEEKDAY

L	σ_L	ρ	L_q	σ_{L_q}	P(D=0)	P(D > 15)	E(D)	Time
5.7717	3.4871	0.9421	4.2687	3.3107	0.0830	0.7196	29.7112	0901
5.6667	3.7333	0.9117	4.2122	3.5065	0.1224	0.6816	29.1890	1001
4.4134	3.5675	0.8977	3.6973	3.4526	0.1022	0.6280	52.4505	1101
4.6337	3.5765	0.8657	3.2525	3.2993	0.1835	0.5823	23.1971	1201
1.5350	2.2064	0.4872	0.7578	1.7268	0.6281	0.1915	6.2655	1301
1.6696	1.9715	0.5978	0.7158	1.5217	0.5217	0.2125	6.5206	1401
1.2572	1.6353	0.5095	0.4444	1.1594	0.6208	0.1482	4.4380	1501
0.8660	0.9972	0.3434	0.0441	0.3119	0.9019	0.0107	0.4848	1601
1.1206	1.2339	0.6778	0.5799	0.9880	0.3220	0.3516	13.3192	1701
1.2384	1.4377	0.6734	0.7013	1.2053	0.3264	0.3607	14.7195	1801
1.3067	1.5318	0.6805	0.7638	1.3073	0.3193	0.3691	15.5306	1901
1.3348	1.5813	0.6809	0.7917	1.3599	0.3189	0.3712	15.8650	2001
1.0908	1.4405	0.6101	0.6041	1.2040	0.3896	0.3170	12.9659	2101
0.6600	1.0762	0.4593	0.2935	0.8203	0.5404	0.2134	7.8451	2201
0.4377	0.8020	0.3619	0.1490	0.5350	0.6379	0.1544	5.2036	2301
0.1222	0.3852	0.1311	0.0176	0.1740	0.8687	0.0474	1.4545	2400

AMOSIST MD Schedule

2 0800-1100
1 1100-1200
2 1200-1530
3 1530-1630
1 1630-2330

LEGEND

L Expected System Size
σ_L Standard Deviation of System Size
ρ Probability a Server is Busy
L_q Expected Queue Size
σ_{L_q} Standard Deviation of Queue size
P(D=0) Probability No Delay
P(D > 15) Probability Delay > 15 min.
E(D) Expected Delay

V. CONCLUSIONS

A. VALIDATION AND SENSITIVITY

The computer model was validated by forcing "steady state" solutions to be achieved. This can be easily done by assuming a constant mean arrival rate, a constant number of servers and a constant mean service rate. In so doing, it became apparent that the model is sensitive to several parameters.

1. Early Arrivals

The effect of arrivals before the system is open for business cannot be overemphasized. The effect of the early arrivals is intuitive in that if the system begins congested, then it must spend a good deal of time alleviating this congestion. Arrivals during the initial periods will normally experience much longer delays than if no patients arrived early. Within the model, early arrivals is an input parameter.

2. AMOSIST MD Schedule

As the output tables indicate, during normal operation the major system congestion occurs in the AMOSIST MD queue. This is attributable to the percentage of the arrival stream which is directed to the AMOSIST MD and to the augmentation of this arrival stream by referrals from the AMOSIST as well as the number of AMOSIST MDs available. This model is very sensitive to the AMOSIST MD schedule and a large expected delay can be seen when few servers are scheduled during peak periods. In actual clinic operation the AMOSIST MD arrival rate is affected by those patients who request to see an AMOSIST MD and also by those AMOSISTs who refer a higher percentage of their patients. The percentage of arrivals going directly to the AMOSIST MD and the referral rate from AMOSISTs to AMOSIST MDs are inputs to the model.

B. SUMMARY

The Acute Minor Illness Clinic at Silas B. Hays Hospital can be successfully modelled as a multi-channel, time dependent queueing system and approximate solutions to desired queue and delay statistics can be found using numerical methods.

The computerized version of the model is capable of handling much larger systems and could be expanded to handle more types of servers. As previously mentioned, the present version with system size limited to 15 is used interactively with the user supplying input and seeing output statistics within 30 seconds.

A general method of solution for multi-channel, time dependent queueing systems has been presented. This model could find ready application to any number of similar queueing systems presently unsolved. The procedure presented is sufficiently general so that a wide variety of, and variance in, input parameters may be easily handled thereby enhancing the model's applicability.

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Thesis 155086
V1513 Van Asdlen
c.1 A numerical solution
for time dependent, multi-channel queues and an
application to the acute
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